Holographic Noise in Atomic Systems and Optical Cavities

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Holographic noise workshop
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Outline

1. Introduction

Outline

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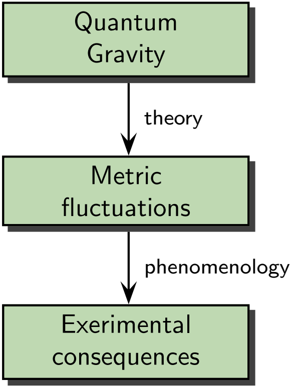
1. Introduction
2. Metric fluctuations in atomic systems
   - The basic equations
   - Equivalence principle
   - Decoherence
   - Spreading of wave packets
3. Metric fluctuations in in optical cavities
4. Summary and outlook

Outline

General remarks I

two–step approach / situation



- **First step:** Derivation of metrical noise from fundamental principles
  - Existence of some metrical noise is generally accepted
  - May perhaps be more general (torsional noise, noise of non–metricity, Finslerian noise, ...)
- **Second step:** search for all consequences of metrical fluctuations (not only holographic noise)
  - search for most promising experiments
  - does not depend on how holographic noise is derived or founded

General remarks II

Geometric noise is **universal**

- geometric noise should appear in all physical systems
  - optical interferometers
  - optical cavities
  - atomic interferometer
  - atoms with spin
  - atomic, molecular energy levels
- all should be consistent with existing data / measurements
- should not depend on temperature, on charge, ...
- characteristics ?

It is not enough to see this noise in one system. **To experimentally show the universality is mandatory for the proof of its existence.**

Classes of experiments

Quantum Gravity

↓

Metric fluctuations

↓

Experimental consequences
- fundamental noise
- effective eqns of motion (nonlocality)
- decoherence
- modified dispersion relation

↙ ↘

photon sector
- interferometry
- optical resonators
- blurring of starlight
- light with polarization
- ...

matter sector
- elementary particles
- bound systems (atoms, molecules)
- particles with spin
- BEC, entangled systems
- ...

Classes of experiments

Much has been done, but this needs more theory:

- calculation of all experimental consequences of space–time fluctuations
- then carrying through all promising experiments

It is important that this noise is **seen in more than one physical system**

Nothing has be seen until now, except perhaps in GEO600

Classes of experiments

Much has been done, but this needs more theory:

- calculation of all experimental consequences of space–time fluctuations
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Is part of the big task: Search for new physics

- violation of Universality of Free Fall
- violation of Universality of Gravitational Redshift
- violation of Lorentz Invariance
- modified gravitational field
- fundamental noise (provides **new window**)

Some history I

space–time foam, fuzzy space–time, space–time fluctuations
goes back to Hawking 1973, Bekenstein 1975, Wheeler 1982

- **Ellis, Hagelin, Nanopoulos & Srednicki 1984** decoherence due to fluctuating metric, neutron interferometry
- **Percival & Strunz 2000** Influence of stochastic metric fluctuations on atom interferometry
- **Power & Percival 2000** Decoherence of wave packets from conformal space–time fluctuations, modified Schrödinger equation for density matrix
- **Amelino–Camelia 2000** Saleker–Wigner argument, random–walk, modified dispersion

  yields general Brownian motion ansatz $S_{\rm sf}^{(\alpha,\gamma)}(\nu) = \zeta \frac{\Lambda}{c} \left(\frac{l_{\rm Planck}}{\Lambda}\right)^{\alpha} \left(\frac{\nu}{c/\Lambda}\right)^{\gamma}$
- **Ng & van Dam 2000** Distance measurement by clocks (based on Saleker–Wigner argument) $\delta g \sim (l_{\rm Planck}/l)^{\frac{2}{3}}$

Some history II

- **Ng 2002** Holographic principle $(l/\delta l)^3 \leq (l_{\text{Planck}}/l) \Rightarrow \delta g \sim (l_{\text{Planck}}/l)^{\frac{2}{3}}$.
  Idea: holographic principle follows from space–time fluctuations
  Relation to quantum computing
  Influence on dispersion relations, decoherence of light phase, UHECR, non–locality, ...
- **Hu & Verdaguer 2002, 2008** Axiomatic approach: Einstein–Langevin equations, application to backreaction problems and black hole fluctuations
- **Ford 2003 – 2008** No specific model, luminosity fluctuations, line broadening, angular blurring, black hole fluctuations
- **Aloisio, Galante, Grillo, Liberati, Lucio & Mendez 2006** no specific scenario, relation to modified dispersion
- **Wang, Bonifacio, Bingham & Mendonca 2006, 2009** Conformal fluctuations and decoherence of quantum particle, effect for very large masses $\sim 10^{19}$ a.m.u.
- **Hogan 2008** Holographic noise in GEO600

This talk

Recent work at ZARM

Influence of space–time fluctuations on quantum systems

- fundamental noise in optical resonators, Schiller, Lämmerzahl, Müller, Braxmaier, Herrmann & Peters, PRD 2004 (experiment)
- apparent violation of weak equivalence principle, Göklü & Lämmerzahl, CQG 2008
- decoherence, Breuer, Göklü & Lämmerzahl, CQG 2008
- spreading of wave packets, Göklü, Lämmerzahl, Camacho & Macias, 2009

our motivation ...

Bremen Drop Tower of ZARM

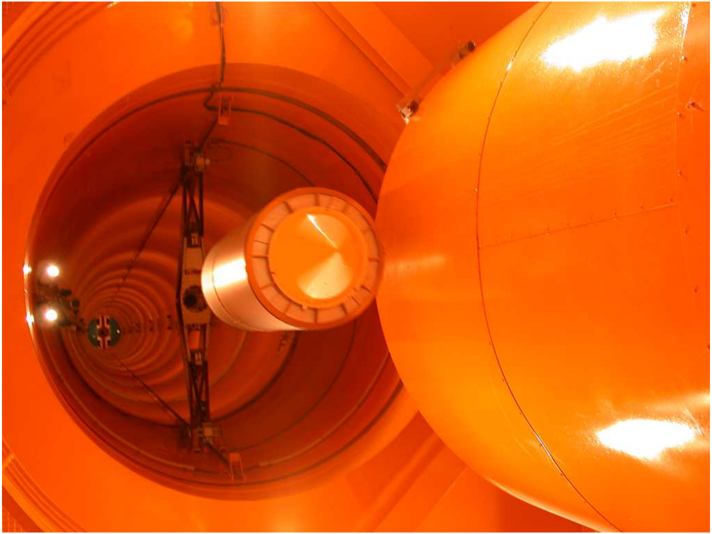
Tower 146 m

drop tube
110 m

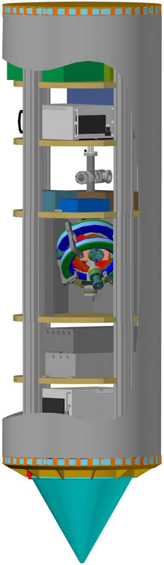
free fall time
$= 4.7$ s

deceleration
$\sim 30\ g$

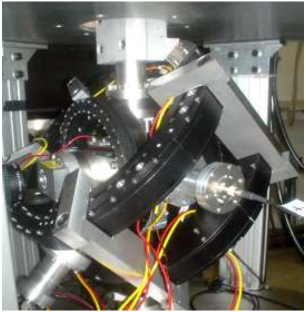
Bremen Drop Tower of ZARM



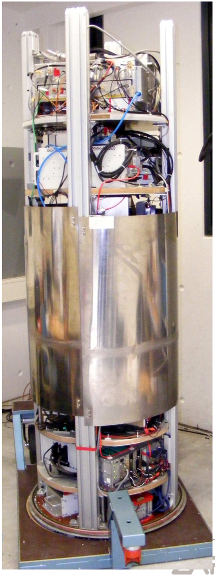
BEC in microgravity



design of capsule



vacuum chamber

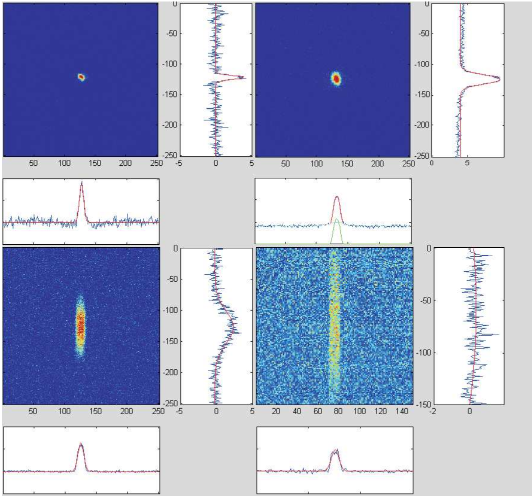


capsule

BEC in microgravity – long free evolution

50 ms

100 ms



500 ms

1000 ms

10^4 atoms, 1 s free evolution time (not possible on ground)

BEC in microgravity – long free evolution

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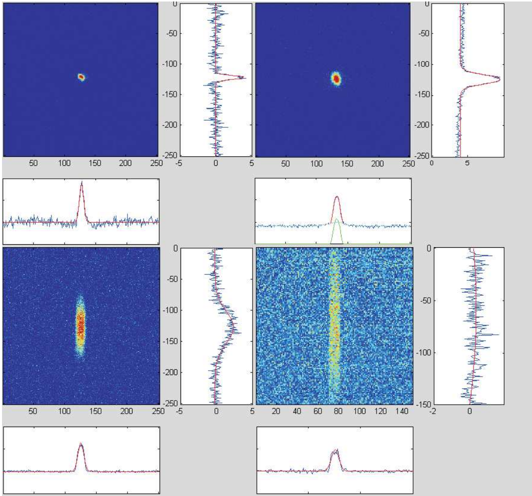

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1000 ms

10^4 atoms, 1 s free evolution time (not possible on ground)

Outline

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The basic equations

The model

- Klein–Gordon equation

$$g^{\mu\nu} D_\mu D_\nu \varphi + m^2 \varphi = 0 \,, \qquad D = \partial + \{ \therefore \}$$

- Fluctuating metric

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \,, \qquad |h_{\mu\nu}| \ll 1$$

- noise

$$\langle h_{\mu\nu}(x) \rangle_{\text{st}} = \gamma_{\mu\nu} \,, \qquad \delta^{\rho\sigma} \langle h_{\mu\rho}(x) h_{\nu\sigma}(x) \rangle_{\text{st}} = \sigma^2_{\mu\nu}$$

- small amplitude of fluctuations
- frequency might be large
- wavelength might be small
- we do not require the $h_{\mu\nu}$ to obey a wave equation

The basic equations

Approximations

- Weak field up to second order $\tilde{h}^{\mu\nu} = h^{\mu\rho} h_\rho{}^\nu$
- Relativistic approximation of metric and quantum field (á la Kiefer and Singh)

$$\begin{aligned} H\psi &= -({}^{(3)}g)^{\frac{1}{4}} \frac{\hbar^2}{2m} \Delta_{\text{cov}} \left(({}^{(3)}g)^{-\frac{1}{4}} \psi' \right) + \frac{m}{2} \left(\tilde{h}^{00}_{(0)} - h^{00}_{(0)} \right) \psi \\ & \quad - \frac{1}{2} \left\{ i\hbar\partial_i, h^{i0}_{(1)} - \tilde{h}^{i0}_{(1)} \right\} \psi \end{aligned}$$

manifest hermitean w.r.t. flat scalar product

- only second order terms do not vanish by averaging

Outline

Short wavelength

Spatial average

- spatial average

$$\langle A\rangle_{\rm s}(x) := \frac{1}{V_x}\int_V A(y)d^3{\rm y}$$

- short wavelength of fluctuations: V small $\quad\Rightarrow\quad \langle\psi\rangle_{\rm s}(x) = \psi(x)$
- spatial average of Schrödinger equation

$$H = \frac{1}{2m}\left(\delta^{ij} + \alpha^{ij}(x)\right)p_i p_j + \alpha_0$$

  with $\alpha(x) = \langle\tilde{h}^{ij} - h^{ij}\rangle_{\rm s}(x)$
- $\alpha^{ij}(x)$: small variation w.r.t. ${\rm x}$, fluctuations w.r.t. t.
- decompose $\alpha^{ij}(x) = \tilde{\alpha}^{ij}(x) + \gamma^{ij}(x)$ with $\langle\gamma^{ij}\rangle_{\rm t} = 0$

- $\tilde{\alpha}^{ij}(x)$ acts like an anomalous inertial mass tensor

Space–time fluctuations

Fluctuation model

- $\alpha^{ij} \leftrightarrow$ spectral noise density of fluctuations
- particular model:

$$\tilde{\alpha}^{ij}(x) = \frac{1}{V_x}\int_{V_x} \tilde{h}^{ij}(\mathrm{x}, t) d^3\mathrm{x} = \frac{1}{V_x}\int_{1/V_x} (S^2(\mathrm{k}, t))^{ij} d^3\mathrm{k}$$

- model: power law spectral noise densitiy

$$(S^2(\mathrm{k}, t))^{ij} = (S^2_{0n})^{ij} |\mathrm{k}|^n \quad \Rightarrow \quad \alpha^{ij}(x) = (S^2_{0n})^{ij} \lambda_p^{-(6+n)}$$

with $\dim(S^2_{0n})^{ij} = \mathrm{length}^{3+\frac{n}{2}}$

- $\lambda_p =$ length scale of particle $= \lambda_{\mathrm{Compton}}$
- $V_x \sim \lambda_p^3$

Space–time fluctuations

Fluctuation model

- assumption: $S_{0n} \sim l_{\rm Planck}^{3+\frac{n}{2}}$, then

$$\alpha^{ij}(x) \sim \left(\frac{l_{\rm Planck}}{l_{\rm Compton}}\right)^{\beta} a^{ij}(x)\,, \qquad \beta = 6+n\,, \;\; a^{ij}(x) = \mathscr{O}(1)$$

- effective Hamiltonian

$$H = \frac{1}{2m}\left(\delta^{ij} + \left(\frac{l_{\rm Planck}}{l_{\rm Compton}}\right)^{\beta} a^{ij}(x)\right) p_i p_j = \frac{1}{2m}\left(\delta^{ij} + \frac{\delta m^{ij}(x)}{m}\right) p_i p_j$$

  δm^{ij} = anomalous inertial mass tensor, depends on particle
- δm^{ij} leads to violation of Universality of Free Fall (Haugan 1979)

- $\beta = \frac{1}{2}$ $\leftrightarrow$ random walk
- $\beta = \frac{2}{3}$ $\leftrightarrow$ holographic noise

Result

Result

metric fluctuations $\Rightarrow$ anomalous inertial mass $\rightarrow$ apparent violation of UFF

- alternative route for violation of UFF and LLI

Example

for Cesium and Hydrogen:

$$\eta_{\beta=1} = 10^{-17}\,, \quad \eta_{\beta=2/3} = 10^{-12}\,, \quad \eta_{\beta=1/2} = 10^{-9}$$

$\beta = \frac{1}{2}$ already ruled out.

(Göklü & C.L. CQG 2008)

are there transverse effects?

Outline

Decoherence

The model

- model as above

$$H = \frac{1}{2m}\left(\delta^{ij} + \tilde{\alpha}^{ij} + \gamma^{ij}(t)\right) p_i p_j$$

- discuss now the influence of γ^{ij} and neglect $\tilde{\alpha}^{ij}$
- neglect small x–dependence

Noise model

- isotropic fluctuations $\gamma^{ij}(t) = \sigma\delta^{ij}\xi(t)$
- white noise $\langle\xi(t)\rangle = 0$, $\langle\xi(t)\xi(t')\rangle = \delta(t - t')$
- $\dim\sigma = \text{time} = \tau_{\mathrm{c}}$

- practically no influence from colored noise
- $\gamma^{ij}(t)$ random process

Decoherence

Master equation

- stochastic Schrödinger equation in interaction picture

$$i\hbar\frac{d}{dt}\mid\tilde{\psi}\rangle = \tilde{H}_\gamma\mid\tilde{\psi}\rangle\,,\qquad \tilde{H} = e^{\frac{i}{\hbar}H_0 t}H_\gamma e^{-\frac{i}{\hbar}H_0 t}$$

with random Hamiltonian $\tilde{H}_\gamma$ with $\langle\tilde{H}_\gamma\rangle_{\mathrm{t}} = 0$

- averaging over fluctuations $\Rightarrow$ averaged density matrix

$$\tilde{\rho}(t) = \langle\mid\tilde{\psi}\rangle\langle\tilde{\psi}\mid\rangle$$

- master equation for averaged density matrix to second order in the fluctuations

$$i\hbar\frac{d}{dt}\tilde{\rho} = -\frac{i}{\hbar}\int_0^t\langle[\tilde{H}_\gamma(t),[\tilde{H}_\gamma(t'),\tilde{\rho}(t)]]\rangle dt'$$

Decoherence

Markovian master equation

- in Schrödinger picture

$$i\hbar \frac{d}{dt}\rho(t) = [H_0, \rho(t)] + i\hbar(\mathscr{D}\rho)(t)$$

with

$$(\mathscr{D}\rho)(t) = -\frac{1}{2}[V, [V, \rho(t)]] \qquad \text{with} \qquad V = \frac{\sqrt{\tau_c}}{\hbar}\frac{\mathrm{p}^2}{2m}$$

- master equation is in Lindblad form $\Rightarrow$ defines a completey positive quantum–dynamical semigroup
- energy is conserved

- $\mathscr{D}$ is the dissipator

Decoherence

Decoherence time

- solution of master equation in momentum space

$$\rho(\mathrm{p},\mathrm{p}',t) = \exp\left(-\frac{i}{\hbar}\Delta E t - \frac{(\Delta E)^2\tau_\mathrm{c}}{2\hbar^2}t\right)\rho(\mathrm{p},\mathrm{p}',0)$$

decoherence time

$$\tau_\mathrm{D} = \frac{2\hbar^2}{(\Delta E)^2\tau_\mathrm{c}} = 2\left(\frac{\hbar}{\Delta E\tau_\mathrm{c}}\right)^2\tau_\mathrm{c}$$

- for $\tau_\mathrm{c} = t_\mathrm{Planck}$

$$\tau_\mathrm{D} = \frac{10^{13}\ \mathrm{s}}{(\Delta E/\mathrm{eV})^2}$$

- too large for being observable
- may change for BECs (Breuer, Göklü & C.L. 2009)

Outline

Spreading of wave packets

Model

- dynamics: same model as above

$$H = H_0 + V(x)\,, \qquad V(x) = \mathscr{O}(h\partial\partial h, \partial h \partial h)$$

- V is Gaussian random function

$$\langle V(x) \rangle = 0\,, \qquad \langle V(x), V(x') \rangle = V_0^2 \delta(t - t') g(\mathrm{x} - \mathrm{x}')$$

longer calculations ...

The spreading

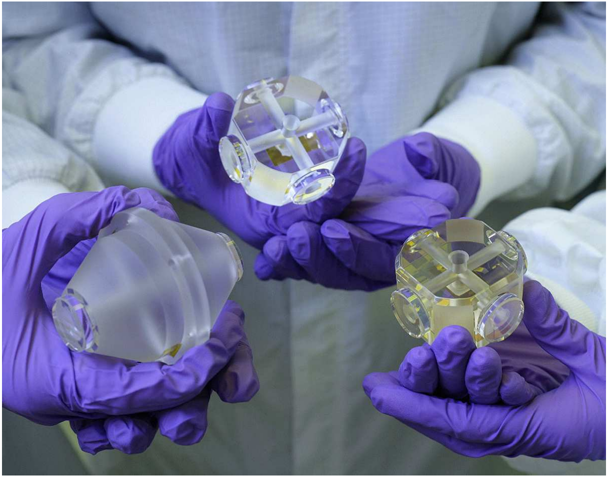
for Gaussian correlation and Gaussian initial wave packet

$$\langle x^2(t) \rangle = \underbrace{\sigma^2 + \frac{\hbar^2}{4m^2\sigma^2} t^2}_{\text{free evolution}} + \underbrace{\frac{\sigma_{px}}{m} t}_{\text{diffusion}} + \underbrace{\frac{V_0}{3\sqrt{2\pi} m^2 a^3} t^3}_{\text{superdiffusion}} \qquad (1)$$

(Göklü, C.L., Camacho & Macias, in preparation)

Outline

Optical resonators



A. Peters

Optical resonators

A. Peters

Optical resonators

M. Tobar

Metric fluctuations in cavities

Ansatz for the strain noise spectrum (Amelino–Camelia PRD 2000)

$$\left(\frac{\Delta L}{L}\right)^2 = \int S_{\rm sf}(\nu) d\nu \qquad \text{with} \qquad S_{\rm sf}(\nu) = \zeta \frac{\Lambda}{c} \left(\frac{L_{\rm Pl}}{\Lambda}\right)^\beta \left(\frac{\nu}{c/\Lambda}\right)^\gamma ,$$

$\Lambda =$ length characteristic of experimental setup
Specification for two random–walk hypotheses:

$$\begin{aligned} S_{\rm sf}^{\beta=1,\gamma=-2} &= 5 \cdot 10^{-27} \zeta_{\rm rw1} \left(\frac{\mathrm{m}}{\Lambda}\right)^2 \left(\frac{\mathrm{Hz}}{\nu}\right)^2 \mathrm{Hz}^{-1} \\ S_{\rm sf}^{\beta=2,\gamma=-2} &= 7 \cdot 10^{-62} \zeta_{\rm rw2} \left(\frac{\mathrm{m}}{\Lambda}\right)^3 \left(\frac{\mathrm{Hz}}{\nu}\right)^2 \mathrm{Hz}^{-1} , \end{aligned}$$

Devices

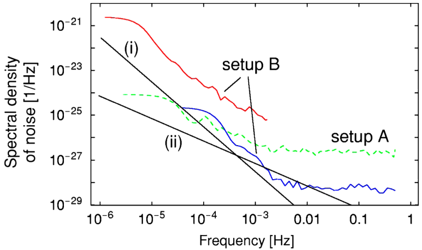
- low frequencies and small devices are preferred setups
- Optical resonators: access to μHz range.

data of a frequency comparison between two optical resonators has been analyzed

Metric fluctuations in cavities

measured quantity $= \nu_2 - \nu_1$

$$\delta\left(\frac{\nu_2-\nu_1}{\nu_1}\right) = \left(\frac{\delta L_2^{\rm sf}}{L_2} - \frac{\delta L_1^{\rm sf}}{L_1}\right) + \left(\frac{\delta L_2^{\rm phys}}{L_2} - \frac{\delta L_1^{\rm phys}}{L_1}\right) + \left(\frac{\delta L_2^{\rm lock}}{L_2} - \frac{\delta L_1^{\rm lock}}{L_1}\right) + \ldots = S_{\rm sf} + S_{\rm phys} + S_{\rm lock} + \cdots$$



setup A: resonators parallel in different cryostats

setup B: two cavities orthogonally in same cryostat

half of total noise represents upper bound to $S_{\rm sf}$.

Comparison with RW ansätze

$$\zeta_{\rm rw1} \leq 2\cdot 10^{-13} \qquad \zeta_{\rm rw2} \leq 4\cdot 10^{20}\,.$$

Parameters are of order 1: rules out RW hypothesis 1 (Schiller et al PRD 2004).
For identification of effect one has to use many materials

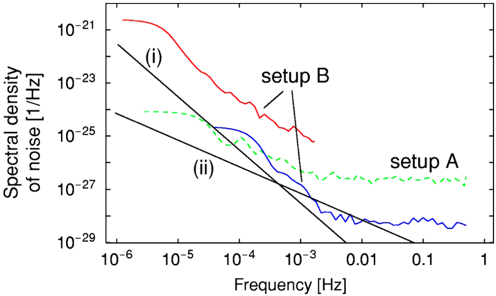
Holographic noise in cavities

Holographic noise

- Spectral noise density for holographic scenario

$$S_{\mathrm{sf}}(\nu) = \zeta \frac{\Lambda}{c} \left(\frac{L_{\mathrm{Pl}}}{\Lambda} \right)^{\beta} \qquad \beta = \frac{2}{3}$$

- for cavity $S_{\mathrm{sf}}(\nu) \sim 10^{-33}\ \mathrm{Hz}^{-1}$



Outline

Summary and outlook

Summary

Metric (holographic) noise appears as

- apparent violation of weak equivalence principle
- decoherence
- additional spreading of wave packets

Reqired

- to proof universality
- to discrimination with other noise surces
- to show cosistency

Outlook

- description of more experiments (effect on spin or helicity, ...)
- effects on BECs (long evolution time – free fall of BECs)

Spin and space–time fluctuations (roughness)

- Geodesic equation for static spherically symmetric metric

$$\left(\frac{dr}{ds}\right)^2 = \frac{1}{g_{tt}g_{rr}}\left(E^2 - g_{tt}\left(\epsilon + m\frac{L^2}{r^2}\right)\right)$$

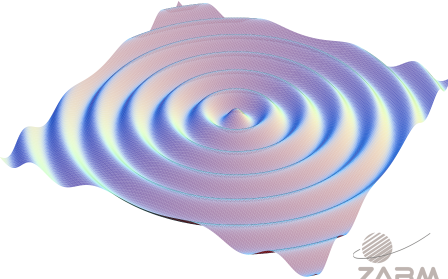
- Assuming $g_{rr} = 1/g_{tt}$ and $g_{tt} = 1 + h\cos(kr)$ (everywhere C^∞)
  Then for radial motion ($L = 0$)

$$\left(\frac{dr}{ds}\right)^2 = E^2 - 1 - h\cos(kr)$$

  Can be solved by elliptic function
- for $h \ll 1$

$$r = \sqrt{E^2 - 1}s + h\frac{\sin(\sqrt{E^2 - 1}ks)}{2\,(E^2 - 1)\,k}$$

  like zitterbewegung



Spin and space–time fluctuations

Kretschmann scalar

$$K = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} \sim \frac{g''_{tt}}{g_{tt}} \sim hk^2 \frac{\cos(kr)}{1 + h\cos(kr)}$$

Consequence

- For $k \to \infty$: solution of geodesic equation approaches straight line
- Kretschmann scalar $\to \infty$

$\Rightarrow$ point particles do not see fluctuating curvature
particles with spin are sensitive to curvature $a = R(\cdot, u, S, u)$

  - may be of importance in atomic interferometry, spectroscopy, ...
  - if fluctuations are of quantum gravity origin $\to$ estimates

- needs to be compared with analysis of Dirac equation in fluctuating space-time metric

Göklü & C.L. in preparation

One final remark

- I think one should look for experiments with other physical systems, e.g., atoms, BECs, ...
- Then one has more possibilities to check consistency with existing data for the other experiments

Thank you !

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- Center of Excellence QUEST
- German Aerospace Agency DLR